

# Transversely Driven Charge Density Waves and Striped Phases of High- $T_c$ Superconductors: The Current Effect Transistor

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(February 1, 2008)

We show that a normal (single particle) current density  $J_x$  *transverse* to the ordering wavevector  $2k_F\hat{\mathbf{z}}$  of a charge density wave (CDW) has dramatic effects both above and *below* the CDW depinning transition. It exponentially (in  $J_x$ ) enhances CDW correlations, and exponentially suppresses the longitudinal depinning field. The intermediate longitudinal I-V relation also changes, acquiring a *linear* regime. We propose a novel “current effect transistor” whose CDW channel is turned on by a transverse current. Our results also have important implications for the recently proposed “striped phase” of the high- $T_c$  superconductors.

PACS: 71.45.Lr, 64.60.Ht

While the equilibrium properties of charge density waves [1] (as well as other disordered periodic media) are by now fairly well understood, recent attention has focussed on their far less well understood *non-equilibrium* dynamical properties. [2]- [5]

All classical treatments of CDW's to date have included only the local displacement  $u(\mathbf{r}, t)$  of the CDW, which is related to the electron density via  $\rho(\mathbf{r}, t) = \rho_0 + \rho_1 \cos(2k_F(z + u(\mathbf{r}, t)))$ , as the only important hydrodynamic mode of the problem.

In this Letter we will argue that while such an approach is correct for the statics, the total electron density  $\rho$  is crucial for the correct description of the *non-equilibrium* dynamics. We will present a dynamical model that incorporates this additional mode and explore its consequences.

This additional mode allows current to flow even when the CDW itself is stationary. The effects are particularly dramatic when the current flows *perpendicular* to  $\mathbf{q}_0 = 2k_F\hat{\mathbf{z}}$ , the ordering wavevector of the CDW. Once this transverse current  $J_x$  exceeds a *crossover* value  $J_c$ , it makes the CDW much more ordered. The correlation length of a three dimensional (3d) CDW along the direction ( $\hat{\mathbf{x}}$ ) of the transverse current obeys

$$\xi_x(J_x) \approx \begin{cases} \xi_L, & \text{for } J_x < J_c, \\ \xi_L \frac{J_c}{J_x} e^{2(J_x - J_c)/J_c}, & \text{for } J_x > J_c, \end{cases} \quad (1)$$

where  $\xi_L$  is the “Larkin length” (see below) of the CDW in zero transverse current, which is finite due to the random pinning of the CDW by impurities.

For directions perpendicular to the transverse current the correlation lengths also grow exponentially with  $J_x$ , but with *precisely half* the growth rate:

$$\xi_{\perp}(J_x) \approx \xi_L \left( \frac{J_c}{J_x} \right) e^{(J_x - J_c)/J_c}, \quad \text{for } J_x > J_c, \quad (2)$$

The “critical” current  $J_c$  in the above expressions is:

$$J_c = \sigma_{\infty} E_T(0) (k_F \xi_L) (\rho_n / \rho_{cdw}) \times O(1) \quad (3)$$

where  $E_T(J_x = 0)$  is the threshold electric field along  $2k_F\hat{\mathbf{z}}$  necessary to depin the CDW, and  $\sigma_{\infty}$  the conductivity in that direction for  $E_z \gg E_T(0)$ . For typical NbSe<sub>3</sub> samples  $J_c \sim 10^3 - 10^4$  Amps/cm<sup>2</sup>. These quite high values can be greatly reduced by increasing disorder (lowering  $\xi_L$ ).

In addition to being directly observable by X-ray scattering measurements of the CDW correlation length, these exponential dependences of the correlation lengths on  $J_x$  also have striking consequences for the I-V characteristics along  $\hat{\mathbf{z}}$ . If we apply a “longitudinal” electric field  $E_z$  along  $\hat{\mathbf{z}}$  while maintaining a fixed  $J_x$ , we find the (zero temperature) threshold depinning field  $E_T(J_x)$  required to first make the CDW move is given by

$$E_T(J_x) = \begin{cases} E_T(0), & \text{for } J_x < J_c, \\ E_T(0) \left( \frac{J_c}{J_x} \right)^{3/2} e^{-2(J_x - J_c)/J_c}, & \text{for } J_x > J_c. \end{cases} \quad (4)$$

Correspondingly, the shape of the high velocity part of the I-V is also drastically modified. We find that for longitudinal fields  $E_z$  in the range,  $E_T(J_x) \ll E_z \ll E_T(0)(J_x/J_c)^2$ , the I-V is *Ohmic* (i.e. linear) with the longitudinal conductivity given by  $\sigma_z(J_x) = \sigma_{\infty} (1 - (J_c/J_x) \times O(1))$ . For the largest fields  $E_z \gg E_T(0)(J_x/J_c)^2$ ,  $J_z(E_z)$  crosses over to the  $J_x = 0$  result:  $J_z(E_z) = \sigma_{\infty} E_z \left( 1 - c \sqrt{E_T(0)/E_z} \right)$ . [2] All of our results for the longitudinal I-V characteristics in the presence of a transverse current are summarized in Fig.1.

Our results suggest a novel “current effect transistor” (CET), in which the threshold field *along*  $\mathbf{q}_0$  is controlled by a current driven *transversely* to  $\mathbf{q}_0$ .

All of the above results should also apply to the recently proposed “striped phase” of the high- $T_c$  cuprate oxide superconductors, [6] since the macroscopic symmetries of that system (i.e., unidirectional, lattice-incommensurate modulation of the charge density) are the same as those of a CDW, as is the ubiquitous presence of disorder. [7]

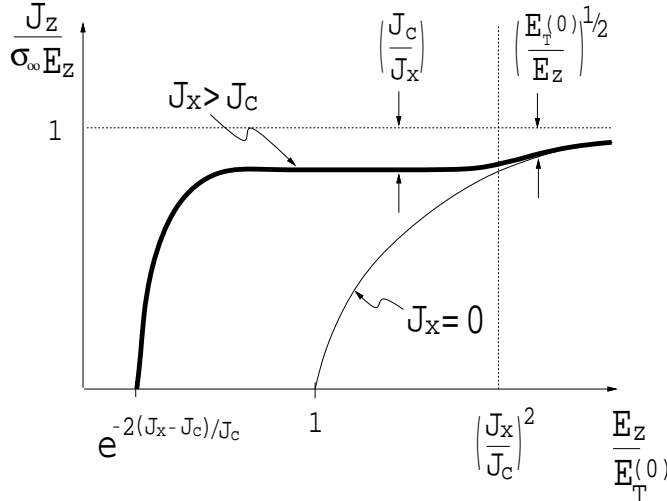


Fig.1:(a) Schematic I-V curve illustrating new intermediate Ohmic high velocity regime that exists for transverse currents  $J_x > J_c$ .

We now present our model and derive these results. General principles dictate that there are two types of long wavelength hydrodynamic variables: Goldstone modes associated with broken symmetries, and conserved variables. In CDW’s, the phonon mode  $u$  associated with the breaking of translational symmetry is the sole Goldstone mode, while the total electron density  $\rho$  is a conserved variable. Contrary to the naive expectation,  $\rho$  is *not* determined solely by the compression of the CDW (i.e.  $\delta\rho \neq -\rho_{cdw}\partial_x u$ ). This is in direct analogy with ordinary 3d crystals, for which the density of vacancies and interstitials (or equivalently the total particle density) must be included in the proper hydrodynamic description [8].

Although the total charge density  $\rho$  is a conserved field, in CDW’s, because of long-range Coulomb interactions [9], it does *not* constitute a *slow* hydrodynamic mode, unlike neutral systems, e.g. the transverse smectic phase of a driven vortex lattice [5].

In contrast, in *driven* CDW’s one *must* consider total charge density dynamics to even obtain the correct equation of motion, to which we now turn.

We consider a pinned CDW with an ordering wavevector  $\mathbf{q}_0 = 2k_F\hat{\mathbf{z}}$  and explore the consequences of a “normal” current  $\mathbf{J}$ . Our main conceptual point is that, in contrast to all previous treatments such a current makes even the *stationary* CDW a highly *non-equilibrium* system in which the  $\hat{\mathbf{J}} \rightarrow -\hat{\mathbf{J}}$  symmetry is broken. Lacking

the spatial inversion symmetry along the direction of such a single-particle current, general symmetry principles therefore dictate that the equation of motion for  $u(\mathbf{r}, t)$  *must* admit and therefore *will* contain non-equilibrium terms that break this inversion symmetry. The most important of these is the “convective” term  $\tilde{\mathbf{v}} \cdot \nabla u$ . The striking effects of this should be experimentally observable.

We explore these in a transverse geometry, in which the charge current  $\mathbf{J}$  flows *transversely* to the CDW ordering wavevector  $2k_F\hat{\mathbf{z}}$ , i.e.  $\mathbf{J} = J_x\hat{\mathbf{x}}$ . The hydrodynamics of a CDW in the presence of such transverse current  $J_x$  is described by

$$\gamma(\partial_t + \tilde{\mathbf{v}} \cdot \nabla)u(\mathbf{r}, t) = K\nabla^2 u + F(\mathbf{r}, u) + e\rho_{cdw}E_z, \quad (5)$$

where  $E_z$  is the externally imposed electric field along  $\mathbf{k}_F$  and  $\rho_{cdw}$  is the average CDW electron number density. We take the quenched random force  $F(\mathbf{r}, u)$  in Eq.5 to be Gaussian distributed with zero mean and  $\overline{F(\mathbf{r}, u)F(\mathbf{r}', u')} = \Delta(u - u')\delta(\mathbf{r} - \mathbf{r}')$ , where the overbar denotes a disorder average. The function  $\Delta(u)$  is periodic with the CDW lattice spacing,  $a = 2\pi/2k_F$ .

Because Galilean symmetry is broken by the underlying lattice and quenched disorder the “convective” coupling  $\tilde{\mathbf{v}}$  is different from (but probably on the order of) the “normal” electron velocity  $\mathbf{v} = \mathbf{J}/\rho_n e$ , where  $\rho_n$  is the “normal” electron density. For simplicity of notation we have taken  $K_x = K_y = K_z = K$ , however all of our results can be trivially extended to anisotropic elasticity.

The essential difference between our model Eq.5 and the commonly used Fukuyama-Lee-Rice (FLR) [1] model is the “convective”  $\tilde{v}_x \partial_x u$  term, which physically arises because a tipped ( $\partial_x u \neq 0$ ) “layer” of the CDW deflects the transverse normal current downward by an angle  $\theta \sim \partial_x u$ , leading to a reaction force back on the CDW, proportional to  $\partial_x u$  and the normal current  $J_x$ .

This “convective” term leads to a velocity-dependent crossover length scale  $L_c(\tilde{v}_x) = K/(\gamma\tilde{v}_x)$ . On length scales  $L_x$  (along  $x$ )  $< L_c$ , the convective term is unimportant and the CDW behaves as though it is in equilibrium; [10] for  $L_x > L_c$ , non-equilibrium effects set in.

In spatial dimensions  $d < 4$ , even arbitrarily weak pinning destroys the translational order of the CDW for length scales longer than the so-called Larkin length scale  $\xi_L$ . [1,11] On this length scale the disordering effect of the random pinning on  $u$  just balances the “elastic” forces that try to keep the CDW ordered.

Clearly, for vanishing or small transverse currents (small  $\tilde{v}_x$ ) the standard FLR-Larkin (FLRL) calculation of  $\xi_L$ , which ignores  $\tilde{v}_x$ , applies, giving in 3d  $\xi_L = K^2 a^2 / \Delta(0)$ .

Given the two length scales  $\xi_L$  and  $L_c(\tilde{v}_x)$ , one can define a crossover transverse velocity  $\tilde{v}_c$  by  $\xi_L = L_c(\tilde{v}_c)$ , giving  $\tilde{v}_c = K/(\gamma\xi_L) = \Delta(0)/(\gamma K a^2)$ . Taking  $\tilde{v}_x \approx J_x/(\rho_n e)$ , we can obtain a crossover *current* density

$J_c = \rho_n e K / (\gamma \xi_L)$ . Using the standard FLR result for the longitudinal threshold electric field  $E_T = K a / (\rho_{cdw} e \xi_L^2)$  and the large-field limit of the longitudinal CDW conductivity  $\sigma_\infty = e^2 \rho_{cdw}^2 / \gamma$ , we obtain Eq.3, with  $J_c$  expressed in terms of experimental observables. For  $J_x < J_c$ , the equilibrium FLRL calculation of  $\xi_L$  is valid and the CDW translational correlation length is independent of the transverse current  $J_x$ .

Interesting transverse current dependent phenomena occur for  $J_x > J_c$ . In this regime, we can calculate the new CDW correlation length  $\xi_\perp(\tilde{v}_x)$  in the  $\perp$ -direction by asking how big in the direction perpendicular to  $x$  the system would have to be before the mean-squared fluctuations  $\overline{\langle u^2 \rangle}$  in the position of the CDW exceed  $a^2$ . For a finite system of width  $L_\perp$ , in  $d < 4$  dimensions, the mean-square phonon fluctuations  $\overline{\langle u^2(\mathbf{r}) \rangle} = \int_{q_\perp > L_\perp^{-1}} dq_x d^{d-1} q_\perp \overline{\langle |u(\mathbf{q})|^2 \rangle}$  can be calculated by spatially Fourier transforming Eq.5 and looking for the static solution for the spatial Fourier transform field  $u(\mathbf{q})$ .

For a system with  $L_\perp \leq \xi_\perp(\tilde{v}_x)$ , the  $u$  fluctuations remain smaller than the lattice spacing  $a$ , and so we can replace  $F(\mathbf{r}, u)$  with its  $u = 0$  value. The static equation of motion then reads

$$(i\gamma \tilde{v}_x q_x + K q^2) u(\mathbf{q}) = F(\mathbf{q}; u = 0), \quad (6)$$

where  $F(\mathbf{q}; u = 0)$  is the spatial Fourier transform of  $F(\mathbf{r}; u = 0)$ . Solving this for  $u(\mathbf{q})$ , and then computing  $\overline{\langle |u(\mathbf{q})|^2 \rangle}$  from the result, we obtain

$$\overline{\langle |u(\mathbf{q})|^2 \rangle} = \frac{\Delta(0)}{\gamma^2 \tilde{v}_x^2 q_x^2 + K^2 q^4}, \quad (7)$$

Inserting this into expression for  $\overline{\langle u(\mathbf{r})^2 \rangle}$  and defining  $\xi_\perp(\tilde{v}_x)$  as the value of  $L_\perp$  at which  $\overline{\langle u(\mathbf{r})^2 \rangle} = a^2$  we obtain an implicit equation for  $\xi_\perp(\tilde{v}_x)$

$$(\xi_L / \xi_\perp(\tilde{v}_x))^{4-d} = f(\xi_\perp(\tilde{v}_x) / L_c(\tilde{v}_x)) \quad (8)$$

where  $f(y) = 2(4-d) \int_1^\infty dx x^{d-2} / (g_+ g_-^{1/2} + g_+^{1/2} g_-)$ , with  $g_\pm(x, y) = x^2 + y^2/2 \pm y(x^2 + y^2/4)^{1/2}$ .

Evaluating the integral in  $d = 3$  in the  $J_x \ll J_c$  and  $J_x \gg J_c$  limits and using  $\tilde{v}_x \approx J_x / (\rho_n e)$  gives Eq.2 for  $\xi_\perp(J_x)$ , which matches the  $J_x = 0$  Larkin length at  $J_x = J_c$ , i.e.,  $\xi_\perp(J_c) \approx \xi_L$ . The correlation length  $\xi_x(J_x)$  along the transverse current  $J_x$  ( $\tilde{v}_x$ ) is easily computed from  $\xi_\perp(J_x)$  to be given by Eq.1.

Physically, the exponential dependence of the CDW correlation length for  $J_x > J_c$  arises due the strong suppression of the CDW roughness by the transversely moving normal charge carriers, which “stiffen” the CDW elasticity by momentum transfer with it. The transverse convective term lowers the upper critical dimension (below which the random pinning disorders the CDW) from  $d_{uc}(\tilde{v}_x < \tilde{v}_c) = 4$  to  $d_{uc}(\tilde{v}_x > \tilde{v}_c) = 3$ .

The strong exponential dependence of the Larkin length on a transverse single particle current  $J_x$  has a number of striking consequences which should be experimentally observable (aside from being directly probed in X-ray scattering). It is easy to show that at low temperatures and weak disorder, the threshold *longitudinal* field (along the ordering wavevector  $2k_F \hat{\mathbf{z}}$ ) is given by  $E_T(J_x) = E_T(0) (V_L(0)/V_L(J_x))^{1/2}$ , where  $E_T(0) = K a / (e \rho_{cdw} \xi_L^2)$  is the  $J_x = 0$  longitudinal threshold electric field and  $V_L(J_x)$  is the transverse current dependent Larkin volume. This decay of the longitudinal threshold field  $E_T$  with Larkin volume is physically associated with the  $\sqrt{N}$  statistic reduction in the strength of the random pinning force, when averaged over the correlation volume  $V(J_x) = \xi_\perp^2(J_x) \xi_x(J_x)$ . Using our results for  $\xi_{x,\perp}(J_x)$ , Eq.1,2 in the expression for  $E_T(J_x)$  above, we obtain Eq.4.

We now turn to the effects of the transverse current  $J_x$  for  $E_z$  well above the longitudinal threshold field  $E_T(J_x)$ . Perturbing about the uniformly moving CDW state  $u_0 = v_z t$ , we seek a solution to Eq.5 of the form  $u(\mathbf{r}, t) = v_z t + \delta u(\mathbf{r}, t)$ , where  $v_z$  is the mean velocity of the CDW, to be determined self-consistently, and  $\delta u(\mathbf{r}, t)$  a zero mean fluctuation (assumed small) about this uniformly moving state. Inserting this Ansatz into Eq.5, expanding for small  $\delta u$ , and averaging over the disorder, we have  $v_z = \gamma^{-1} e \rho_{cdw} E_z + \delta v_z$ , with

$$\gamma \delta v_z \approx \overline{\partial_u F(\mathbf{r}, u) \delta u(\mathbf{r}, t)} \Big|_{u=v_z t}, \quad (9)$$

where we have used the facts that  $\overline{F(\mathbf{r}, v_z t)} = \overline{\nabla^2 \delta u} = 0$ . We can obtain  $\partial_u F \delta u$  from the equation of motion for  $\delta u(\mathbf{r}, t)$ . To leading order in  $\delta u$ , that equation reads:

$$(\gamma \partial_t + \gamma \tilde{\mathbf{v}} \cdot \nabla - K \nabla^2) \delta u = \sum_{q_n} F(\mathbf{r}, q_n) e^{i q_n v_z t}, \quad (10)$$

where we have used the fact that  $F(\mathbf{r}, u)$  is a periodic function of  $u$  and  $q_n = n q_0$ , with  $n \in \mathbb{Z}$ .

Solving Eq.10 for  $\delta u$  and using it inside Eq.9 for  $\delta v_z$  we obtain

$$\gamma \delta v_z \approx - \sum_{q_n} \int \frac{d^d q}{(2\pi)^d} \frac{\Delta_{q_n} q_n (v_z q_n - \tilde{\mathbf{v}} \cdot \mathbf{q})}{\gamma^2 (v_z q_n - \tilde{\mathbf{v}} \cdot \mathbf{q})^2 + (K q^2)^2} \quad (11)$$

We can verify a posteriori that the wavevector integral in this expression is dominated by  $q_z \ll q_n$ . This together with  $\tilde{\mathbf{v}} \sim \mathbf{v}$  implies  $\tilde{\mathbf{v}} \cdot \mathbf{q} \approx \tilde{v}_x q_x$ , so:

$$\gamma \delta v_z \approx - \int \frac{d^d q}{(2\pi)^d} \frac{2 \Delta_0 \tilde{v}_x q_0 (q_1 - q_x)}{\gamma^2 \tilde{v}_x^2 (q_1 - q_x)^2 + K^2 q^4} \quad (12)$$

where we have defined  $q_1 \equiv q_0 v_z / \tilde{v}_x$  and kept only  $n = 1$ ; higher random force harmonics ( $n > 1$ ) do not qualitatively change the results.

We can asymptotically evaluate the integral in Eq.12 in the limits: (i)  $q_x^* \ll q_1$  and (ii)  $q_x^* \gg q_1$ , where

$q_x^*$  is the characteristic value of  $q_x$  that dominates the integral. A simple analysis shows that in (i)  $q_x^* = \sqrt{\gamma v_z q_0 / K}$ . The condition  $q_x^* < q_1$  then leads to the constraint  $E_z / E_T(0) > \text{Max}\{1, (J_x / J_c)^2\}$ . Ignoring  $q_x$  relative to  $q_1$  in Eq.12 we find that the SCF result [2] holds at these highest longitudinal fields:  $\delta v_z / v_z = -(E_T(0) / E_z)^{(4-d)/2} \times O(1)$ . For  $J_x < J_c$ , this is the only regime.

For  $J_x > J_c$ , we find that a new regime (regime (ii)) exists, when the longitudinal fields  $E_z / E_T(0) < (J_x / J_c)^2$ . Here  $q_1$  is subdominant to  $q_x$  and the integral above can be evaluated by a Taylor expansion in  $q_1$ . Changing variables  $q_x - q_1 \rightarrow q_x$  and expanding in  $q_1$  we find

$$\frac{\delta v_z}{v_z} \approx -8\Delta_0 q_0^2 K^2 \int \frac{d^d q}{(2\pi)^d} \frac{q_x^2 q^2}{[\gamma^2 \tilde{v}_x^2 q_x^2 + K^2 q^4]^2} \quad (13)$$

The integral on the right hand side is *finite* in the infrared and the ultraviolet for  $1 < d < 4$ . All higher terms in this  $q_1$  expansion are likewise finite. Evaluating the integral in  $d = 3$  we obtain

$$\frac{\delta v_z}{v_z} \approx -\frac{\Delta q_0^2}{K \gamma \tilde{v}_x} \times O(1) = -\frac{\tilde{v}_c}{\tilde{v}_x} \times O(1) \quad (14)$$

A more detailed analysis [7] shows that the above expansion breaks down as  $E_T(\tilde{v}_x)$  is approached from above. This can be seen simply by assuming that  $\tilde{v}_z$  in Eq.5 is of order the  $v_z$  we found above, and using that result in a calculation of  $\langle \delta u(\mathbf{r}, t)^2 \rangle$ . We find that this approaches  $a^2$ , invalidating our small  $\delta u$  expansion, for  $E_z \sim$  the threshold field  $E_T(\tilde{v}_x)$  found earlier. This breakdown signals the pinning of the CDW for  $E_z < E_T(\tilde{v}_x)$ .

This conclusion is consistent with our earlier observation that  $J_c$  is a *crossover* (as opposed to *critical*) transverse current, and with similar results for the related transverse smectic phase of a driven vortex lattice. [5] Our results for the transversely driven CDW I-V characteristics are summarized in Fig.1.

Another observable quantity sensitive to transverse current is the Fourier transform  $I(\mathbf{r})$  of the structure function:

$$\overline{\langle \rho_{2k_F}^*(\mathbf{r}) \rho_{2k_F}(0) \rangle} = \begin{cases} e^{-r/\xi_L}, & \text{for } J_x < J_c, \\ r_{\perp}^{-\eta(J_x)} g_{\eta} \left( \frac{r_{\perp}^2}{x \xi_L} \frac{J_x}{J_c} \right), & \text{for } J_x \gg J_c, \end{cases}$$

where  $\eta(J_x) = O(1) \times J_c / J_x$  and  $g_{\eta}(z)$  is a scaling function readily calculated to lowest order in weak disorder. It is important to keep in mind that the expression above is only valid for length scales smaller than the Larkin length.

On scales larger than the Larkin length, the small  $u$  expansion of the random force is invalid. However, using the functional renormalization group developed in Ref. [5], we can show that  $u$  fluctuations are logarithmic for both  $J_x < J_c$ , and for  $J_x > J_c$ . In the former case, the phonon correlation function  $C(r_{\perp}, x)$  (which grows linearly with  $r$  for  $r < \xi_L$ ) first (at the Larkin scale) crosses

over to the familiar super-universal (for  $2 < d < 4$ ) logarithmic dependence on  $r$  [12]. At scales larger than  $L_c(\tilde{v}_x)$  the correlation function is also logarithmic (but with non-universal coefficient), now as a consequence of the fact that non-equilibrium effects have lowered the upper-critical dimension to  $d_{uc} = 3$ . [5] For  $J_x > J_c$  the intermediate regime is absent and there is a direct crossover at the Larkin scale to the logarithmic growth associated with non-equilibrium effects.

Finally we note that our results, in particular Eq.4, might have an interesting device application: the “current effect transistor” (CET). Imagine a channel made of e.g., NbSe<sub>3</sub>. [13] At low  $T$ , and longitudinal fields below the threshold field  $E_T$ , such a channel is insulating. For  $E_z > E_T$  the CDW depins and the channel becomes highly conducting. Our analysis demonstrates that this *longitudinal* threshold field  $E_T(J_x)$  is a strong function of the *transverse* current  $J_x$  for  $J_x > J_c$ , as summarized by Eq.4. One can therefore turn on the CDW transport by a transverse current  $J_x$  which depins the CDW by depressing the longitudinal threshold field  $E_T(J_x) \sim e^{-2(J_x - J_c)/J_c}$ . Though theoretically interesting, this “device” would almost certainly be too slow and low gain to be practical.

In conclusion, we have argued that previous treatments of sub-threshold CDW dynamics have ignored an important “permeation” (total charge density) mode. We demonstrate that this mode radically changes the CDW dynamics in the transverse current geometry.

L.R. acknowledges discussions with A.C. Neto and support by the NSF DMR-9625111 and the A.P. Sloan Foundation; J.T. was supported by the NSF DMR-9634596.

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